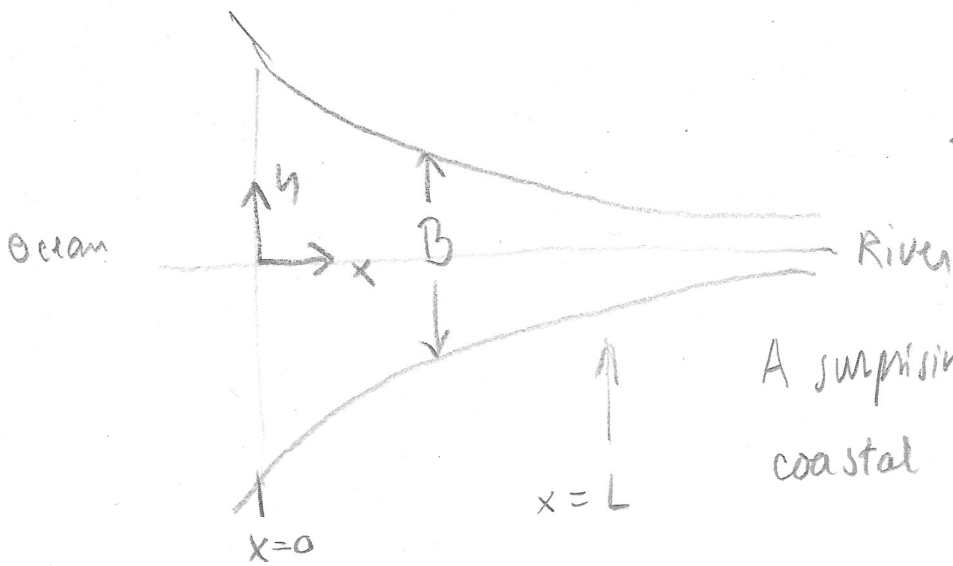
(3)

$\boxed{\text{Trumpet-Shaped Estuary}}$ frictional, SW

$\boxed{\text{x mom}}$ $\boxed{u_t + g\eta_x + Ru = 0}$

$\boxed{\text{mass}}$ $B\eta_t + (HBu)_x = 0$ ← modified to allow for changing channel width



$B = B_0 \exp(-x/L)$

A surprisingly common shape for coastal plane estuaries.

Rewrite $\boxed{\text{mass}}$, for $H = \text{const}$, noting $\frac{B_x}{B} = \frac{-1}{L}$

$$\Rightarrow \boxed{\eta_t + Hu_x - \frac{H}{L}u = 0} \ \boxed{\text{mass}}$$

Assume solutions of the form

$u = \mathrm{Re}\{\mathcal{U} \exp i(kx - \omega t)\}$

$\eta = \mathrm{Re}\{E \exp i(kx - \omega t)\}$

$\mathcal{U}$ & E are complex constants, k can be complex.

No reflected wave, Converging channel will increase u, but friction will decrease it!

Plugging in:

subititute

$$-i\omega \mathcal{U} + igk E + R\mathcal{U} = 0$$

$$-i\omega E + iHk\mathcal{U} - \frac{H}{L}\mathcal{U} = 0 \Rightarrow E = \frac{(iHk - H/L)}{i\omega}\mathcal{U}$$

$$\Rightarrow \left(-i\omega + \frac{igk(iHk - H/L)}{i\omega} + R\right)\cancel{\mathcal{U}} = 0$$

$$\Rightarrow \quad \omega^2 + i\omega R - gHk^2 - igHk/L = 0 \qquad \text{define } c^2 = gH$$

$$\frac{\omega^2}{c^2} - k^2 + i\left(\frac{\omega R}{c^2} - \frac{k}{L}\right) = 0 \quad (*)$$

This is a quadratic in k that we can solve, but a special simple case is when k has no complex part, meaning the <u>amplitude</u> of η and u are constant along the channel. If u_{max} is just below the critical stress for sediment erosion, the channel morphology will not change, a state called "morphological equilibrium".

This happens where the imaginary part of

(*) vanishes $\Rightarrow$ $\dfrac{\omega R}{c^2} = \dfrac{k}{L}$

or $L = \dfrac{c^2}{R}\dfrac{k}{\omega} = \dfrac{c}{R}$ (for $\dfrac{\omega}{k} = c$)

so this particular solution has its

channel length set by H & $\mathcal{U}$

(as they affect $c = \sqrt{gH}$ and $R = C_d\,\mathcal{U}/H$)

For $R = 1.5 \times 10^{-4}\ s^{-1}$

and $c = \sqrt{g \cdot 20\,m} = 14\ m\,s^{-1}$

$L = \dfrac{c}{R} = \dfrac{14}{1.5 \times 10^{-4}} \cong 10^5\ m = 100\ km$ ✓